# MOTION OF A CONDENSED PARTICLE IN A CHANNEL WITH INJECTION 

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Consideration is given to problems associated with modeling of the motion of a condensed particle in a channel with injection with allowance for the action of different force factors (the hydrodynamic-drag force, the Saffman lift force, and the thermophoresis force). From the results of the numerical modeling, the author draws conclusions on the degree of influence of different force factors on the pattern of motion of the particle.

Introduction. Flow in a channel with permeable walls is used as a mathematical model of flow of the products of disintegration of a solid propellant in a rocket-engine combustion chamber and reflects the most significant aspect of the process: mass supply on the source side of a burning charge surface [1-3]. Injection models the burning of the interior channel surface (strong injection) or its thermal destruction (weak injection). Processes associated with the heating of the propellant and the disintegration of its components and their chemical reaction occur in a thin surface layer and are disregarded in this model.

Metal additions in the form of a highly dispersed powder (mainly aluminum), which are part of many types of modern mixture solid propellants, are necessary for attainment of the required level of energy characteristics and damping of uncontrolled acoustic fluctuations of the parameters of a working medium in the combustion chamber. The quality of modeling of the fluxes of combustion products is largely dependent on the accuracy of description of their properties.

In this work, consideration is given to problems associated with construction of a model of interaction between a condensed particle and a carrier flow in a channel with injection. From the results of the numerical modeling, we draw conclusions on the degree of influence of different force factors (the hydrodynamic-drag force, the Saffman lift force, and the thermophoresis force) on the pattern of motion of the particle. We propose an approach which makes it possible to simplify realization of a numerical model of motion of a dispersed impurity in a channel with permeable walls.

Carrier Phase. Let us consider quasideveloped flow of a viscous incompressible fluid in an infinite plane slit of width $h$ on both sides of which we have injection with a prescribed intensity (Fig. 1). The condition of quasidevelopment of the flow implies that the flow characteristics referred to the maximum velocity in the cross section change only slightly with channel length [2]:

$$
\frac{h}{u_{\mathrm{m}}}\left|\frac{d u_{\mathrm{m}}}{d x}\right| \ll 1 .
$$

In injection, such a flow is established behind the region of the inlet portion in fairly long channels.
Let us bring the $x$ axis of the Cartesian system into coincidence with the lower channel wall. We assume that the spreading of the fluid is symmetric about the plane $x=0$.

We select the channel width $h$ as the characteristic scales for variables with the dimensions of length and rate of injection of the fluid from the upper wall $v_{\mathrm{w} 2}$ (it is assumed that $v_{\mathrm{w} 2} \neq 0$ ) and the temperature of the upper channel wall $T_{\mathrm{w} 2}$ as the characteristic scales for variables with the dimensions of velocity and temperature. The characteristic parameter of the problem is the Reynolds number $\operatorname{Re}=v_{\mathrm{w} 2} h / v$.
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Fig. 1. Flow in a channel with injection.
Let us assume that the longitudinal velocity of the fluid changes linearly along the coordinate $x$, whereas the transverse velocity component and the temperature are only dependent on the coordinate $y$ :

$$
\begin{equation*}
u=x \varphi(y), \quad v=g(y), \quad T=\vartheta(y) . \tag{1}
\end{equation*}
$$

Using (1) and eliminating the pressure from the momentum equations in projections onto the $x$ and $y$ axes by cross differentiation, we may show that the modeling of a viscous incompressible fluid flow in a channel with injection is reduced to integration of the equations

$$
\begin{gather*}
g^{(4)}-\operatorname{Re}\left(g g^{\prime \prime \prime}-g^{\prime} g^{\prime \prime}\right)=0  \tag{2}\\
\vartheta^{\prime \prime}-\operatorname{Pe} g \vartheta^{\prime}=0 \tag{3}
\end{gather*}
$$

The dynamic and thermal problems are separated for the incompressible fluid, and (3) is solved after the integration of (2). Boundary conditions for Eqs. (2) and (3) are set on the lower and upper channel wall and have the following form:

$$
\begin{aligned}
& g=\omega, \quad g^{\prime}=0, \quad \vartheta=\theta \text { at } y=0 \\
& g=-1, \quad g^{\prime}=0, \quad \vartheta=1 \text { at } y=1
\end{aligned}
$$

The solution of the boundary-value problem is constructed based on the Newton linearization method with subsequent iterations by nonlinearity.

Condensed Particle. For high values of the Reynolds number determining the relative flow past a particle, the dominant influence is exerted by nonlinear iteration effects, whereas the influence of nonstationary effects (frequently defined as hereditary ones) in the gas phase turns out to be very slight. Thus, the ratio of the forces - the Archimedes force, the additional-mass force, and the Basset force - to the drag force has the order of the ratio of the gas density to the particle density [4].

The rotation acquired by the particle under the action of the gradient of carrier-flow velocity in many flows is not so large as to be responsible for the particle's transverse motion occurring under the influence of the Magnus force [4].

In the interphase-interaction model, allowance is made for the action of the hydrodynamic-drag force, the Saffman lift force, and the thermophoretic force. The motion of the particle is described by the following system of equations:

$$
\begin{gather*}
\frac{d x_{\mathrm{p}}}{d t}=u_{\mathrm{p}},  \tag{4}\\
\frac{d y_{\mathrm{p}}}{d t}=v_{\mathrm{p}},  \tag{5}\\
\frac{d u_{\mathrm{p}}}{d t}=\frac{9 \mu}{2 \rho_{\mathrm{p}}^{2} r_{\mathrm{p}}^{2}} \phi_{\mathrm{d}}\left(u-u_{\mathrm{p}}\right)+\frac{1}{m_{\mathrm{p}}}\left(f_{\mathrm{S} x}+f_{\mathrm{tx}}\right),  \tag{6}\\
\frac{d v_{\mathrm{p}}}{d t}=\frac{9 \mu}{2 \rho_{\mathrm{p}} r_{\mathrm{p}}^{2}} \phi_{\mathrm{d}}\left(v-v_{\mathrm{p}}\right)+\frac{1}{m_{\mathrm{p}}}\left(f_{\mathrm{S} y}+f_{\mathrm{t} y}\right) . \tag{7}
\end{gather*}
$$

The drag coefficient of the particle is represented in the form

$$
C_{\mathrm{d}}=\frac{24}{\operatorname{Re}_{\mathrm{p}}} \phi_{\mathrm{d}}\left(\operatorname{Re}_{\mathrm{p}}\right)
$$

The function $\phi_{d}$ allows for the correction for the particle's inertia [4]:

$$
\phi_{\mathrm{d}}\left(\operatorname{Re}_{\mathrm{p}}\right)=1+0.179 \operatorname{Re}_{\mathrm{p}}^{0.5}+0.013 \operatorname{Re}_{\mathrm{p}}
$$

The Reynolds number in relative motion of the particle and the carrier gas is found as

$$
\operatorname{Re}_{\mathrm{p}}=\frac{2 r_{\mathrm{p}} \rho\left|\mathbf{v}-\mathbf{v}_{\mathrm{p}}\right|}{\mu}
$$

The projections of the Saffman lift force onto the axes of the Cartesian coordinate systems are computed from the formulas [5, 6]

$$
\begin{aligned}
& f_{\mathrm{S} x}=C_{\mathrm{S}} \phi_{\mathrm{S}} r_{\mathrm{p}}^{2} \rho \sqrt{v}\left(v-v_{\mathrm{p}}\right) \frac{\partial v / \partial x}{|\partial v / \partial x|}, \\
& f_{\mathrm{S} y}=C_{\mathrm{S}} \phi_{\mathrm{S}} r_{\mathrm{p}}^{2} \rho \sqrt{v}\left(u-u_{\mathrm{p}}\right) \frac{\partial u / \partial y}{|\partial u / \partial y|}
\end{aligned}
$$

Here we have $C_{S}=6.46$, and the function $\phi_{S}$ allows for corrections related to the inertia of the particle, the velocity gradient of the fluid, and the proximity of the wall [7, 8]. In a shear flow, the lift-force coefficient is represented in the following form:

$$
C_{\mathrm{S}}(\alpha, \eta)=6.46 f_{\mathrm{S}}(\alpha, \eta), \quad \alpha=\frac{\operatorname{Re}_{\mathrm{p}}}{\sqrt{\operatorname{Re}_{\mathrm{p}, \mathrm{v}}}}, \quad \eta=y \sqrt{\operatorname{Re}_{\infty}}
$$

The Reynolds number characterizing the degree of nonuniformity of the flow is calculated from the in-plane shear of the velocity $d u / d y$ and the particle size:

$$
\operatorname{Re}_{\mathrm{p}, \mathrm{v}}=\frac{r_{\mathrm{p}}^{2} \rho}{\mu}\left|\frac{d u}{d y}\right|
$$

Specific dependences for the function $\phi_{S}$ have been given in [7, 8].

To compute the components of the thermophoresis force in a continuous regime of flow we use the relations [4, 9]

$$
\begin{aligned}
& f_{\mathrm{tx}}=-12 \pi C_{\mathrm{t}} r_{\mathrm{p}} \rho v^{2} \frac{\lambda / \lambda_{\mathrm{p}}}{1+2 \lambda / \lambda_{\mathrm{p}}} \frac{\partial\left(T / T_{\infty}\right)}{\partial x}, \\
& f_{\mathrm{ty}}=-12 \pi C_{\mathrm{t}} r_{\mathrm{p}} \rho \nu^{2} \frac{\lambda / \lambda_{\mathrm{p}}}{1+2 \lambda / \lambda_{\mathrm{p}}} \frac{\partial\left(T / T_{\infty}\right)}{\partial y} .
\end{aligned}
$$

Here we have $C_{\mathrm{t}}=1.17$.
Taking into account that, by virtue of relations (1), the projections of the lift force and the thermophoresis force onto the $x$ axis are equal to zero, we write Eqs. (6) and (7) as follows:

$$
\begin{gathered}
\frac{d u_{\mathrm{p}}}{d t}=B_{\mathrm{d}} \phi_{\mathrm{d}}\left[x_{\mathrm{p}} \varphi\left(y_{\mathrm{p}}\right)-u_{\mathrm{p}}\right] \\
\frac{d v_{\mathrm{p}}}{d t}=B_{\mathrm{d}} \phi_{\mathrm{d}}\left[g\left(y_{\mathrm{p}}\right)-v_{\mathrm{p}}\right]+B_{\mathrm{S}} \phi_{\mathrm{S}}\left[x_{\mathrm{p}} \varphi\left(y_{\mathrm{p}}\right)-u_{\mathrm{p}}\right] \sqrt{x_{\mathrm{p}}} \frac{\varphi^{\prime}\left(y_{\mathrm{p}}\right)}{\sqrt{\left|\varphi^{\prime}\left(y_{\mathrm{p}}\right)\right|}}+B_{\mathrm{t}} \vartheta^{\prime}\left(y_{\mathrm{p}}\right),
\end{gathered}
$$

where

$$
\begin{gathered}
B_{\mathrm{d}}=\frac{9}{2} \frac{\rho}{\rho_{\mathrm{p}}}\left(\frac{r_{\mathrm{p}}}{h}\right)^{-2} \frac{1}{\operatorname{Re}} ; \\
B_{\mathrm{S}}=\frac{3 C_{\mathrm{S}}}{4 \pi} \frac{\rho}{\rho_{\mathrm{p}}}\left(\frac{r_{\mathrm{p}}}{h}\right)^{-1} \frac{1}{\sqrt{\mathrm{Re}}} ; \\
B_{\mathrm{t}}=-9 C_{\mathrm{t}} \frac{\rho}{\rho_{\mathrm{p}}} \frac{\lambda / \lambda_{\mathrm{p}}}{1+2 \lambda / \lambda_{\mathrm{p}}}\left(\frac{r_{\mathrm{p}}}{h}\right)^{-2} \frac{1}{\mathrm{Re}^{2}} .
\end{gathered}
$$

The coordinates of the particle are determined from Eqs. (4) and (5).
At the initial instant of time, the particle is on the channel wall and is injected into the channel along the normal to its lateral surface; therefore, we have

$$
x_{\mathrm{p}}(0)=x_{\mathrm{p} 0}, \quad y_{\mathrm{p}}(0)= \pm 1, \quad u_{\mathrm{p}}(0)=0, \quad v_{\mathrm{p}}(0)=\mp \chi \text { at } t=0 .
$$

The upper and lower signs are taken for particles escaping from the upper and lower channel walls respectively.
The equations describing the particle's motion are integrated using the implicit Euler method. To complete the gasdynamic characteristics of the fluid at the points lying on the particle's trajectory we use cubic spline interpolation.

Exact Solution for the Case of Strong Injection. When $\mathrm{Re} \rightarrow \infty$, Eq. (2) has an exact solution. In the case $\omega=1$, the distributions of the components of the carrier-medium velocity are found from the relations [1]

$$
\begin{gathered}
u=\frac{\pi}{2}(s+1) x \cos \left(\frac{\pi}{2} y^{s+1}\right), \\
v=-y^{-s} \sin \left(\frac{\pi}{2} y^{s+1}\right)
\end{gathered}
$$

A comparison to the calculated results and the data of physical experiment shows that the exact solution fairly well describes the velocity distribution in a channel with injection for $\mathrm{Re}>80$, including the turbulent regime of flow, if $x<30-50$ [1-3].

With allowance for the action of just the hydrodynamic-drag force, the equation of motion of a Stokes particle in a channel with intense injection acquires the form

$$
\begin{gathered}
\frac{d u_{\mathrm{p}}}{d t}=B_{\mathrm{d}}\left[\frac{\pi}{2}(s+1) x_{\mathrm{p}} \cos \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right)-u_{\mathrm{p}}\right] \\
\frac{d v_{\mathrm{p}}}{d t}=-B_{\mathrm{d}}\left[y_{\mathrm{p}}^{-s} \sin \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right)+v_{\mathrm{p}}\right]
\end{gathered}
$$

The coordinates of the particle are found from Eqs. (4) and (5).
The particle's motion in the transverse (radial) direction is described by the expression

$$
\begin{equation*}
\frac{d^{2} y_{\mathrm{p}}}{d t^{2}}+B_{\mathrm{d}} \frac{d y_{\mathrm{p}}}{d t}+B_{\mathrm{d}} y_{\mathrm{p}}^{-s} \sin \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right)=0 \tag{8}
\end{equation*}
$$

In the axial region (for $y_{p} \rightarrow 0$ ), the nonlinear term allows linearization of the form

$$
y_{\mathrm{p}}^{-s} \sin \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right) \sim \frac{\pi}{2} y_{\mathrm{p}} .
$$

Then Eq. (8) takes the form

$$
\begin{equation*}
\frac{d^{2} y_{\mathrm{p}}}{d t^{2}}+B_{\mathrm{d}} \frac{d y_{\mathrm{p}}}{d t}+\frac{\pi}{2} B_{\mathrm{d}} y_{\mathrm{p}}=0 \tag{9}
\end{equation*}
$$

and its solution is dependent on the sign of the discriminant $\Delta=B_{\mathrm{d}}^{2}-2 \pi B_{\mathrm{d}}$.
The equations of motion of the particle in the longitudinal (axial) direction will be written as

$$
\begin{equation*}
\frac{d^{2} x_{\mathrm{p}}}{d t^{2}}+B_{\mathrm{d}} \frac{d x_{\mathrm{p}}}{d t}-\frac{\pi}{2}(s+1) B_{\mathrm{d}} x_{\mathrm{p}} \cos \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right)=0 \tag{10}
\end{equation*}
$$

Let us linearize the nonlinear term in the axial region:

$$
\frac{\pi}{2}(s+1) x_{\mathrm{p}} \cos \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right) \sim \frac{\pi}{2}(s+1) x_{\mathrm{p}} .
$$

After substituting it into (10), we obtain the equation

$$
\frac{\pi}{2}(s+1) x_{\mathrm{p}} \cos \left(\frac{\pi}{2} y_{\mathrm{p}}^{s+1}\right) \sim \frac{\pi}{2}(s+1) x_{\mathrm{p}}
$$

Noteworthy is the existence of two qualitatively different forms of transverse motion of a particle in the channel with injection. The solution of Eq. (9) for $B_{\mathrm{d}}>2 \pi$ (small particles) is aperiodic in character: the channel axis is an asymptote for the trajectories of particles. The solution of Eq. (9) for $B_{\mathrm{d}}<2 \pi$ (large particles) allows the intersection of the particle's trajectory and the channel axis. The value $B_{\mathrm{d}}=2 \pi$ is the criterion of the particle crossing the channel axis.

Impurity-Concentration Distribution. The concentration of the impurity at the instant of time $t$ is found from the continuity equation written in Lagrange variables:

$$
\begin{equation*}
\left(\frac{x_{\mathrm{p}}}{x_{\mathrm{p} 0}}\right)^{s} n_{\mathrm{p}}\left(x_{\mathrm{p} 0}, y_{\mathrm{p} 0}, t\right) \frac{\partial y_{\mathrm{p}}\left(x_{\mathrm{p} 0}, y_{\mathrm{p} 0}, t\right)}{\partial t} \frac{\partial x_{\mathrm{p}}\left(x_{\mathrm{p} 0}, y_{\mathrm{p} 0}, t\right)}{\partial x_{\mathrm{p} 0}}+1=0 \tag{11}
\end{equation*}
$$

In the region of small particles (for $B_{\mathrm{d}}>2 \pi$ ), the concentration of the dispersed phase has a singularity near the channel axis $\left(n_{\mathrm{p}}\left(\mathbf{r}_{\mathrm{p}}\right) \rightarrow \infty\right.$ for $\left.\mathbf{r}_{\mathrm{p}} \rightarrow 0\right)$. Let us denote a sphere with a radius $r$ and its center on the channel axis by $S(r)$. We compute the number of particles within this sphere:

$$
N(r)=\int_{S} n_{\mathrm{p}} d \mathbf{r} .
$$

Using Eq. (11), we may show, analogously to [10], that

$$
N(r)=C r^{\gamma}+o\left(r^{\gamma}\right)
$$

The exponent $\gamma$ determines the order of the singularity of the concentration of the dispersed phase $(1+\sqrt{2}<\gamma<3$ for $s=0$ and $2 \sqrt{3}<\gamma<3$ for $s=1$ ). The singularity of the concentration of particles for $\mathbf{r}_{\mathrm{p}} \rightarrow 0$ is integrable; the singularity becomes weaker as the particle size increases [10].

Time of Residence of a Particle in the Channel. Let us consider the equation of motion of a particle

$$
\begin{equation*}
u_{\mathrm{p}} \frac{d u_{\mathrm{p}}}{d x}=B_{\mathrm{d}}\left(u-u_{\mathrm{p}}\right) . \tag{12}
\end{equation*}
$$

The above equation allows a solution of the form

$$
\begin{equation*}
u_{\mathrm{p}}=k u, \quad k=\frac{\sqrt{B_{\mathrm{d}}^{2}+4 B_{\mathrm{d}}}-B_{\mathrm{d}}}{2} . \tag{13}
\end{equation*}
$$

Using it, we may evaluate the character of retardation of particles in internal flows whose gasdynamics is similar to the structure of flow in a channel with permeable walls.

Let us carry out evaluation for the time of residence of a particle in the channel portion of flow. We write the equations of motion of a portion of the fluid:

$$
\begin{gathered}
\frac{d x}{d t}=\frac{\pi}{2}(s+1) x \cos \left(\frac{\pi}{2} y^{s+1}\right) \\
\frac{d y}{d t}=-y^{-s} \sin \left(\frac{\pi}{2} y^{s+1}\right)
\end{gathered}
$$

We separate the variables and eliminate the transverse coordinate:

$$
\frac{d x}{x}=\frac{\pi}{2}(s+1) \frac{1-\exp [-\pi(s+1) t]}{1+\exp [-\pi(s+1) t]} .
$$

Integrating from 0 to $t$, we obtain the time of motion of the medium's portion along an arbitrary streamline as a function of the channel length:

$$
t_{\mathrm{f}}=\frac{2}{\pi(s+1)} \ln \left[\frac{L}{x_{0}}-\sqrt{\left(\frac{L}{x_{0}}\right)^{2}-1}\right]
$$

For a fixed channel length, the residence time of a fluid particle on the arbitrary streamline (if $x_{0} \neq 0$ ) is finite, whereas for $x_{0}=0$ the residence time of the medium's portion in the channel becomes infinitely long.

A rough evaluation of the residence time of a condensed particle in the channel with injection is given by the relation

$$
t_{\mathrm{p}}=\frac{t_{\mathrm{f}}}{k}=\frac{2 t_{\mathrm{f}}}{\sqrt{B_{\mathrm{d}}^{2}+4 B_{\mathrm{d}}}-B_{\mathrm{d}}}
$$



Fig. 2. Profiles of the longitudinal (a) and transverse (b) velocity components for $\omega=0$ : 1) $\operatorname{Re}=0$; 2) 10 ; 3) 20 ; 4) $10^{2}$; 5) $10^{3}$.

Calculation Results. The influence of viscosity manifests itself as an insignificant filling of the profile of the longitudinal velocity component in the central part of the channel. Increase in the Reynolds number leads to a decrease in the role of viscous stresses in formation of the flow pattern (Fig. 2). When $\operatorname{Re}>10^{3}$ the solution is virtually independent of the Reynolds number, and when $\omega=1$ (bilateral injection of the same intensity) the velocity distribution is fairly well described by the solution for vortex nonviscous-liquid flow in a channel with injection [1-3].

As the rate of injection from the surface $y=0$ increases, the independence of the solution from the Reynolds number occurs for its lower values (it is equal to approximately 80 for $\omega=1$ ). For low $\omega$ (weak injection from the lower surface), the geometric position of the maximum of the longitudinal-velocity-component distribution approaches the lower wall, and the surface flow itself becomes similar to the flow in the boundary layer on an impermeable surface. The profiles of the transverse component of the velocity vector change over the channel cross section relatively slightly in a fairly wide range of the parameters of the problem.

Equations (4)-(7) are integrated numerically for different ratios of the particle size to the channel width. We set $\operatorname{Re}=10^{4}$ and $\rho / \rho_{\mathrm{p}}=4.5 \cdot 10^{-4}$ in all the calculated variants. The particle trajectories in the channel are given in Fig. 3. The solid curves show the particle trajectories calculated without allowance for the influence of the Saffman force, whereas the dashed curves show those obtained with allowance for the action of the lift force (for $\phi_{S}=1$ ). Different groups of curves (solid and dashed ones) correspond to particles escaping from the upper and lower walls of the channel for the same values of the longitudinal coordinate $\left(x_{\mathrm{p} 0}=5\right)$ and other parameters of the problem. When $\omega=$ 1 (bilateral injection of the same intensity), the results are consistent with the data obtained in [11], where fluid flow in a channel is described based on complete Navier-Stokes equations.

The influence of the lift force on the trajectories of motion of a particle escaping from the upper permeable channel wall is insignificant, on the whole, and becomes lower, the larger the particle size. At the same time, the Saffman force qualitatively changes the trajectories of a particle escaping from the lower channel wall. Further increase in the rate of injection from the lower channel wall (with increase in the value of $\omega$ ) leads to a decrease in the role of the Saffman lift force in formation of the pattern of particle motion (particularly, in short channels $(L<15 h)$ ). The calculation results show that the influence of the corrections (described by the function $\phi_{S}$ ) on the inertia of the particle and the gradient nature of the flow does not lead to a qualitative and quantitative change in the pattern of particle motion. In modeling two-phase flows in channels with injection, we may disregard these corrections, using the value $C_{\mathrm{S}}$ $=6.46$ for the proportionality factor in representation of the Saffman force.

The results of numerical modeling show that allowance for the thermophoresis force for $\theta<2.5$ does not lead to a qualitative and quantitative reconstruction of the pattern of particle motion. Extension of the model to higher $\theta$ values requires that one allow for the temperature dependence of the carrier-medium density.

The data obtained are consistent with analytical evaluations. Since we have $v \ll u$ in the channel, disregarding the component of the drag force in the projection onto the $y$ axis and allowing for (1) (here, $\partial v / \partial x=0$ ), for the ratio of the Saffman force to the drag force we obtain the following estimate:


Fig. 3. Trajectory of motion of a particle in the channel for $\delta=2.5 \cdot 10^{-4}$ (a and b) and $4.0 \cdot 10^{-4}$ (c and d) and $v_{\mathrm{w}}=0(\mathrm{a}$ and c ) and 0.2 ( b and d): 1) particles escape from the upper channel wall, 2) from the lower wall.


Fig. 4. Change in the particle velocity along the channel axis for $\delta=4.0 \cdot 10^{-4}$ for different points of escape of the particle from the lateral channel surface: 1) $x_{\mathrm{p} 0}=1$; 2) 3; 3) 5 .

$$
\frac{f_{\mathrm{S}}}{f_{\mathrm{d}}}=\frac{C_{\mathrm{S}}}{6 \pi} \frac{r_{\mathrm{p}}}{h}\left[\operatorname{Re} x_{\mathrm{p}}\left|\varphi^{\prime}(y)\right|\right]^{1 / 2} \sim \operatorname{Re}_{\mathrm{p}, \mathrm{v}}^{1 / 2}
$$

The flow is low-gradient $(\partial u / \partial y \rightarrow 0)$ in the central part of the channel and near its walls for $\omega=1$; therefore, the Saffman force is smaller than the force of hydrodynamic drag. A substantial influence of the Saffman lift force on the
motion of the impurity would be expected when a boundary layer is formed in the vicinity of the lower channel wall (for $\omega=0$ ).

The relative contribution of the thermophoresis force is found from the relation

$$
\frac{f_{\mathrm{t}}}{f_{\mathrm{d}}}=C_{\mathrm{t}} \frac{2 \lambda / \lambda_{\mathrm{p}}}{1+2 \lambda / \lambda_{\mathrm{p}}} \operatorname{Re} \frac{\vartheta^{\prime}(y)}{x_{\mathrm{p}} \varphi(y)|1-k|}
$$

The influence of the thermophoresis force becomes weaker with distance from the left-hand boundary of the computational domain (the particles are accelerated). The thermophoresis force exerts an appreciable influence on the motion of finely divided particles $(k \sim 1)$ for fairly high temperature gradients.

Figure 4 shows the change in the longitudinal component of the particle velocity along the channel axis (for $\delta=4.0 \cdot 10^{-4}$ ). The dashed line corresponds to the change in the fluid velocity. The given results show that the particle velocity changes virtually linearly along the coordinate $x$ (except for the small initial portion which is the shorter, the smaller the particle size). This circumstance makes it possible to replace the integration of the equation of motion of a particle in the projection onto the $x$ axis by relation (13), which considerably reduces the time of numerical calculation. The same effects also occur in particle motion in a turbulent flow, which will make it possible to use the data obtained for prediction of the motion of an impurity under more complicated conditions.

Conclusions. From the data of the numerical modeling, we have established the influence of the Saffman force and the thermophoresis force on the formation of the pattern of motion of an impurity in a channel with injection as a function of the particle size and the ratio of the rates of injection from the channel walls. The calculation results are important for construction of a model of interaction between a condensed particle and a carrier flow in modeling two-phase flows in the combustion chambers of solid-propellant rocket engines within the framework of the model of interpenetrating continua or the discrete-trajectory approach.

## NOTATION

$B$, coefficient in the representation of the force of action of the carrier flow on a particle; $C$, proportionality factor; $f$, force per unit mass, $\mathrm{N} / \mathrm{kg}$; $g$, function describing the transverse velocity component as a function of the coordinate $y ; h$, channel width, $\mathrm{m} ; k$, coefficient of velocity nonequilibrium of phases in the longitudinal direction; $L$, channel length, $\mathrm{m} ; m$, mass, $\mathrm{kg} ; n$, concentration, $1 / \mathrm{m}^{3} ; N$, number of particles; Pe, Péclet number; $r$, radius, m; r, radius vector, m; Re, Reynolds number; $s$, index of the geometry of flow; $S$, sphere surface; $t$, time, sec; $T$, temperature, K ; $u$ and $v$, velocity components, $\mathrm{m} / \mathrm{sec} ; \mathbf{v}$, velocity vector, $\mathrm{m} / \mathrm{sec} ; x, y$, coordinates, $\mathrm{m} ; \alpha$, correction for the gradient nature of the flow; $\gamma$, exponent; $\delta$, ratio of the particle radius to the channel width; $\Delta$, discriminant of the characteristic equation; $\eta$, transformed transverse coordinate; $\theta$, ratio of the temperatures of the lower and upper channel walls; $\vartheta$, function describing the dependence of the temperature on the coordinate $y ; \lambda$, thermal conductivity, $\mathrm{W} /(\mathrm{m} \cdot \mathrm{K})$; $\mu$, dynamic viscosity, $\mathrm{kg} /(\mathrm{m} \cdot \mathrm{sec}) ; v$, kinematic viscosity, $\mathrm{m}^{2} / \mathrm{sec} ; \rho$, density, $\mathrm{kg} / \mathrm{m}^{3} ; \phi$, functional dependence; $\varphi$, function describing the dependence of the longitudinal velocity component on the coordinate $y ; \chi$, initial velocity nonequilibrium of phases; $\omega$, ratio of the rates of injection from the lower and upper channel walls. Subscripts and superscripts: d, drag; f, fluid particle; m, maximum; p, particle; S, characteristics of the Saffman force; $t$, thermophoresis; v, shear; w, channel wall; $x$ and $y$, projections onto the $x$ and $y$ axes; 0 , initial instant of time; 1 and 2, lower and upper channel walls; ', derivative with respect to the coordinate $y$.

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